

PLASMON AND EXCITON SUPERCONDUCTIVITY
MECHANISMS IN LAYERED STRUCTURES

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16. Abstract Plasmon and exciton superconductivity mechanisms are discussed. Superconductivity in a 3-layer metal-semiconductor-metal and insulator-semimetal-insulator sandwich structure is described in terms of the temperature dependent Green function of the longitudinal (Coulomb) field. The dependences of the superconducting transition temperature on structure parameters were obtained. In a semiconducting film, as a result of interactions of degenerate free carriers with excitons, superconductivity exists only in a certain range of parameters values, and the corresponding critical temperature is much lower than in the plasmon mechanism of superconductivity.					
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Introduction

Interesting results have recently been obtained, in experimental study of superconductivity in nonuniform (heterogeneous) PbTe systems [1,2]. In work [1], superconductivity was observed, with a critical temperature of $T_c \approx 5.3^\circ\text{K}$, in a PbTe semiconductor, onto the surface of which thin, granular films of various metals (Pb, In, Tl, Al, Sn) had been sputtered. Measurements of the magnetic susceptibility of specially treated PbTe specimens containing Pb granules of various sizes, conducted in [2], indicate the presence of superconductivity, right up to a temperature of $T_c \approx 20^\circ\text{K}$. /3

It should be emphasized that the possibility of superconductivity in semiconductor films with granular metallic coatings was predicted in [3]. It was proposed that Cooper pairing of degenerate carriers takes place in the film, by means of exchange with virtual quanta of collective dipole oscillations of electrons in the granules [4].

Together with this, in a heterogeneous semiconductor-metal system, in principle, the so-called "exciton" mechanism of superconductivity can be achieved [5-7]. However, the possibility of a significant increase in the critical superconductivity transition temperature T_c by means of the exciton mechanism was placed in doubt in works [8,9], in connection with covalent instability [8,10] and the development of bound surface states [11,12],¹ as well as with a reduced (by virtue of the large exciton radius) role of local field effects [7,14-17], which play an important part in the phonon mechanism of superconductivity. More than that, due to the finite excitation energy of excitons ω_{ex} , the effective retarded interaction between electrons they cause is of a repulsive nature near the Fermi surface, in the $0 < \omega \leq \omega_{ex}$ energy region [18]. It should also be remembered that an electron-exciton interaction inside a metal is considerably attenuated, due to shielding. As a result of this, metallic films in layered structures of the "sandwich" type, considered by Ginzburg and Kirzhnits [5,6], should be as thin as possible ($d \approx 1.0$ nm). On the other /4

¹We note that, in this case, two-dimensional superconductivity can occur at the Tamm levels [11], through the interaction of electrons with surface (Rayleigh) phonons [13].

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hand, in the Allender-Bray-Bardeen model [7], conduction electrons, penetrating from a metal into a dielectric or semiconductor, screen the Coulomb attraction between electrons and holes. This can lead to a decrease in the binding or break-down energy of exciton states. Thus, in itself, the exciton mechanism of superconductivity evidently is not sufficiently effective. However, as was shown in works [19-21], within the framework of dielectric formalism, the presence of attraction peaks far from the Fermi surface, connected with exchange with virtual excitons, favors an increase of T_c in systems where, due to electron-phonon interaction, superconductivity already exists. (The analysis carried out in [8] showed that an increase in superconductivity through the exciton mechanism is possible, only in substances where $T_c \lesssim 5^\circ\text{K}$.) Nevertheless, the question of the effectiveness of the exciton mechanism cannot be considered to be finally clarified, since examination of the problem for heterogeneous systems has been carried out, as a rule, on the basis of various model potentials, which approximately describe the image and shielding forces [6,21,22]. /5

At the same time, in semiconductor (semimetallic) films with a metallic coating, the specific nonphonon mechanism of superconductivity, which is caused by the interaction of degenerate free carriers with surface plasma oscillations at the metal-semiconductor boundary [23-25], and which is a two-dimensional analog of the "plasmon" mechanism of superconductivity in many-valley (multiregion) semiconductors and semimetals [26-28] or in transition s-d metals [29-30], may prove to be more effective. The main advantage of the plasmon mechanism over the exciton is that the effective attraction between electrons, in this case, is concentrated in the immediate vicinity of the Fermi surface, in the $0 < \omega \lesssim \Omega_p$ energy region (where Ω_p is the plasma frequency of the electrons in a metal). We also note that shielding does not have a destructive effect on plasmons (in distinction from excitons). Moreover, since the density of states in a semiconductor (semimetal) is considerably less than in a metal, shielding of an electron-plasmon interaction in a film is small, so that the thickness of the latter can be $d \sim 3.0-10.0$ nm. /6

In the present work, by means of the temperature-dependent Green function of the longitudinal (Coulomb) field in three-layered systems, obtained by Romanov [31], which describes the delayed, shielded interaction between electrons, with the contribution of surface excitations taken into account, a detailed examination of plasmon and exciton superconductivity in metal-semiconductor-metal and insulator-semimetal-insulator "sandwiches" is carried out. It is shown that the interaction of degenerate free carriers in a semiconductor film with excitons results in superconductivity, only in a specific region of parameters of the system, and that the corresponding critical temperatures are very much lower than the critical temperatures for plasmon superconductivity.

2. Equation for Gap and Coulomb Green Function in Three-Layer System.

We consider superconductivity in a three-layer system ("sandwich" type), consisting of a film of a degenerate semiconductor or semimetal of thickness d , with a metallic or insulator (semiconductor) coating.

We will proceed from the equation of Gor'kov [32] for the "gap" $\Delta(\vec{p}, \vec{p}')$ in the energy spectrum of a heterogeneous superconductor, as $T \rightarrow T_c$ [24, 33]:

$$\begin{aligned} \Delta(\vec{p}, \varepsilon_\ell; x, x') = & -T \sum_m \int \frac{d^2 p'}{(2\pi)^2} \int dx'' \int dx''' \times \\ & \times G_0(\vec{p}', -\varepsilon_m; x''', x) \Delta(\vec{p}', \varepsilon_m; x''', x'') G_0(\vec{p}', \varepsilon_m; x'', x') \times \\ & \times \{ D_{ph}(\vec{p} - \vec{p}', \varepsilon_\ell - \varepsilon_m; x, x') + e^2 D_c(\vec{p} - \vec{p}', \varepsilon_\ell - \varepsilon_m; x, x') \}. \end{aligned} \quad (2.1)$$

Here, $G_0(x, x')$ is the temperature-dependent Green function of free (nonpaired) electrons, \vec{p} and \vec{p}' are two-dimensional pulses (in the planes of the film), $\varepsilon_\ell = (2\ell + 1)\pi T$ is a quantified odd "frequency," D_{ph} is the Green function of phonons and $D_c(x, x')$ is the temperature-dependent Green function of the longitudinal (Coulomb) field, obtained in [31]. Inside the film, i.e., with $0 \leq x, x' \leq d$, function $D_c(x, x')$, consisting of symmetrical $D_c^{(s)}$ and antisymmetrical $D_c^{(a)}$ parts, has the following form [31]:

$$D_c(\vec{q}, \omega_\ell; x, x') = D_c^{(s)}(\vec{q}, \omega_\ell; x, x') + D_c^{(a)}(\vec{q}, \omega_\ell; x, x'); \quad (2.2)$$

$$\begin{aligned} D_c^{(s, a)}(\vec{q}, \omega_\ell; x, x') = & 4\pi \left\{ \theta^{(s, a)}(\vec{q}, \omega_\ell; x, x') - \right. \\ & \left. - \frac{2\alpha^{(s, a)}(\vec{q}, \omega_\ell; x) \theta^{(s, a)}(\vec{q}, \omega_\ell; 0, x')}{2\alpha^{(s, a)}(\vec{q}, \omega_\ell; 0) + [q \varepsilon_0(i\omega_\ell)]^{-1}} \right\}; \end{aligned} \quad (2.3)$$

where

$$\alpha^{(s, a)}(\vec{q}, \omega_\ell; x) = \frac{1}{d} \sum_{q_\nu^{(s, a)}} \frac{e^{iq_\nu x}}{(q^2 + q_\nu^2) \cdot \varepsilon(\vec{q}, q_\nu, i\omega_\ell)}; \quad (2.4)$$

$$\beta^{(S,A)}(\vec{q}, \omega_e; x, x') = \frac{1}{d} \sum_{q_\nu^{(S,A)}} \frac{e^{i q_\nu x} \cos q_\nu x'}{(q^2 + q_\nu^2) \epsilon(\vec{q}, q_\nu, i\omega_e)} \quad (2.5)$$

$$q_\nu^{(S)} = 2\pi\nu/d; q_\nu^{(A)} = (2\nu+1)\pi/d; (\nu=0, \pm 1, \pm 2, \dots); \quad (2.6)$$

$\omega_e = 2\pi\ell T$ is an even quantified "frequency," $\epsilon_0(i\omega_e)$ is the dielectric permeability of the external medium (without allowance for spatial dispersion), and $\epsilon(\vec{q}, q_\nu, i\omega_e)$ is the dielectric permeability of the film, which takes into account quantization of the transverse pulse of electrons, as a result of their reflection from the interface of the media. We note that, in the limit $d \rightarrow \infty$, the Fourier representation of function $D_0(x, x')$ changes to the longitudinal component $D_{00}(\vec{q}, \omega_e) = \frac{4\pi}{q^2 \epsilon(\vec{q}, i\omega_e)}$ of the Green function of the electromagnetic field in a uniform medium [34]. In this work, in distinction from [6-8, 18-21], we will be interested in superconductivity caused by nonphonon mechanisms in pure form and, therefore, subsequently, we will omit D_{ph} in the equations for the gap. /8

If the heterogeneity of the oscillation field is disregarded, which holds for sufficiently thin films or long wavelengths, when $qd \ll 1$, and the spatial dispersion in the field also is disregarded, which corresponds to disregarding shielding of the interactions, as a result of summation over $q_\nu^{(S,A)}$ in (2.4) and (2.5), with $x=x'=0$, we obtain

$$\alpha^{(S)}(\vec{q}, \omega_e, 0) = \beta^{(S)}(\vec{q}, \omega_e, 0, 0) = \frac{\text{cth}(qd/2)}{2q \epsilon(i\omega_e)} \quad (2.7)$$

$$\alpha^{(A)}(\vec{q}, \omega_e, 0) = \beta^{(A)}(\vec{q}, \omega_e, 0, 0) = \frac{\text{th}(qd/2)}{2q \epsilon(i\omega_e)} \quad (2.8)$$

It follows from this that, with $qd \ll 1$, the main contribution to (2.3) is made by symmetrical modes of oscillation, since, with $qd \gg 1$, the symmetrical and antisymmetrical oscillations make the same contribution. /9

Dimensional quantization effects, which play an important part in the case of an ideally pure, uniform film of constant thickness, with mirror reflecting surfaces, have been examined in

detail in works [24,25], where, in particular, oscillations of the critical temperature T_c were obtained, as a function of electron concentration n_0 (or thickness d) connected with sequential filling of two-dimensional subregions in impulse space.

However, actual thin films, as a rule, are nonuniform, both in thickness and in composition (subblock structure), or they are amorphous and have a rough surface, which scatters electrons in a diffuse manner, which should result in suppression of dimensional quantization. The finite permeability of barriers at interfaces (heterojunctions) leads to the same thing.

In connection with this, we examine a semiconductor (semimetallic) film, in which quantization of the single particle electron spectrum is completely absent (but the spectrum of slightly attenuated collective excitations is discrete). We assume that the electrons inside the film are distributed through the cross section with constant density, on the average, i.e., $G_0(x, x') \equiv G_0(x - x')$ and the coherence length ξ_0 is large, compared with d , so that the parameter of order in the film is uniform: $\Delta(x, x') \equiv \Delta(x - x')$. Moreover, we will consider that the free path length of the carriers ℓ considerably exceeds d , so that the effective coherence length $\xi \sim (\ell/\xi_0 + 1/\ell)^{-1} \gg d$. If the depth of penetration of the surface oscillation field $\lambda \gg d$, function $D_c(x, x')$ can be replaced in equation (2.3) by its value at the surface of the film, at points $x = x' = 0$ or $x = x' = d$. Then, in the absence of spatial dispersion, in accordance with (2.7) and (2.8), we have

$$D_c(\vec{q}, \omega_e; 0, 0) = D_c(\vec{q}, \omega_e; d, d) = \frac{2\pi}{q} \left\{ \frac{c \operatorname{th}(qd/2)}{\epsilon_0(i\omega_e) c \operatorname{th}(qd/2) + \epsilon(i\omega_e)} + \frac{\operatorname{th}(qd/2)}{\epsilon_0(i\omega_e) \operatorname{th}(qd/2) + \epsilon(i\omega_e)} \right\} \quad (2.9)$$

Under conditions $qd \ll 1$ and $\epsilon_0 \gg \epsilon q^2 d/2$, expression (2.9) is significantly simplified:

$$D_c(q, \omega_e; 0, 0) \approx \frac{2\pi}{q \epsilon_0(i\omega_e)} \quad (2.10)$$

By substituting (2.10) in (2.1) and by changing to a Fourier representation of the transverse coordinate $(x - x')$, as a result of integration over the angle between vectors \vec{p} and \vec{p}' in the planes of the film and averaging the gap over the transverse energy of the electrons $\xi_1 = K_1^2/2m^*$ (where m^* is the effective mass), we obtain:

$$\bar{\Delta}(\varepsilon_\ell) = -\frac{2m^*e^2}{\pi K_F} T \sum_m \frac{\bar{\Delta}(\varepsilon_m)}{\varepsilon_0(i|\varepsilon_\ell - \varepsilon_m|)} \left(\frac{d\xi_\perp}{E_F} \right)_0^{\varepsilon_\ell} \int_0^{\xi_\parallel} \frac{K(\sqrt{1-\xi_\parallel/E_F})}{\varepsilon_m^2 + (\xi_\perp + \xi_\parallel)^2} d\xi_\parallel, \quad (2.11)$$

where $\xi_\parallel = (p^2 - K_F^2)/2m^*$, $K_F = (3\pi^2 n_0)^{1/3}$ is the Fermi impulse, $E_F = K_F^2/2m^*$ is the Fermi energy, and $K(t)$ is the total elliptical integral of the first kind ($t \leq 1$). Since $K(\sqrt{1-\xi_\parallel/E_F})$ has a logarithmic characteristic at point $\xi_\parallel = 0$, by changing the order of integration in (2.11) and by removing the continuous function $f(\xi_\parallel)$ from under the sign of the integral over ξ_\parallel , with $\xi_\parallel = 0$, with the relation

$$\int_0^1 dy K(\sqrt{1-y}) = 2 \int_0^1 t K(t) dt = 2 \quad (2.12)$$

taken into account, we reduce equation (2.12) to the form:

$$\bar{\Delta}(\varepsilon_\ell) = -\frac{4m^*e^2}{\pi K_F} T \sum_m \frac{\bar{\Delta}(\varepsilon_m)}{\varepsilon_0(i|\omega_\ell - \omega_m|)} \int_0^{E_F} \frac{d\xi_\perp}{\varepsilon_m^2 + \xi_\perp^2}. \quad (2.13)$$

3. Plasmon Superconductivity

We consider the situation, when the outside medium ("sandwich" covers) is a metal (or semimetal) with a high concentration of degenerate free carriers, and the film is a semiconductor. Let the work function of the metal be less than the work function of the semiconductor, so that injection of electrons into the film takes place at the metal-semiconductor contact. If the thickness of the film is less than the dimensions of the space charge region and the width of the forbidden band is sufficiently small, degeneration of the carriers can occur in the film. We note that the conditions indicated above can, in particular, be satisfied by semiconductor junctions of the HgTe, PbTe, SnTe or $\text{Pb}_x\text{Sn}_{1-x}\text{Te}$ type, with anomalously narrow forbidden bands and low effective masses. (According to [35], a SiTe junction may be of interest, with respect to superconducting properties.)

In the case under consideration, the dielectric permeability of the outer medium $\varepsilon_0(i\omega_\ell)$, with conditions $\omega_\ell > qv_0$ (where v_0 is the Fermi velocity of the electrons in the metal) equals:

$$\varepsilon_0(i\omega_\ell) \cong \varepsilon_\infty \left(1 + \tilde{\Omega}_p^2 / \omega_\ell^2 \right), \quad (3.1)$$

where $\tilde{\Omega}_p = \Omega_p / \sqrt{\varepsilon_\infty}$ is the effective plasma oscillation frequency and ε_∞ is the high frequency dielectric permeability of the lattice. If inequality $K_F v_0 \ll \tilde{\Omega}_p$ is satisfied, expression (3.1) holds true in a broad region of transmitted energies and pulses, and the

equation for the gap (2.13), as $T \rightarrow T_c$ can, with good accuracy be represented in the form:

$$\bar{\Delta}(\varepsilon_\ell) = -4\alpha T_c \sum_m \bar{\Delta}(\varepsilon_m) \frac{(\varepsilon_\ell - \varepsilon_m)^2}{\tilde{\Omega}_p^2 + (\varepsilon_\ell - \varepsilon_m)^2} \int_0^{E_F} \frac{d\xi}{\varepsilon_m^2 + \xi^2}, \quad (3.2)$$

where $\alpha = \frac{m^* e^2}{\pi K_F \varepsilon_\infty}$. By analogy with the Morel-Anderson approximation [36], we will look for a solution of linear integral equation (3.2), in the form of the continuous test function

$$\bar{\Delta}(\varepsilon_\ell) = \Delta_1 \frac{\tilde{\Omega}_p^2}{\varepsilon_\ell^2 + \tilde{\Omega}_p^2} - \Delta_2. \quad (3.3)$$

There follows from the boundary conditions, with $\ell=0$ and $\ell \rightarrow \infty$, to within the terms $\sim (\pi T_c / \tilde{\Omega}_p)^2$,

$$\Delta_1 - \Delta_2 = -4\alpha T_c \sum_m \left\{ \Delta_1 \frac{\tilde{\Omega}_p^2}{(2m+1)^2 \pi^2 T_c^2 + \tilde{\Omega}_p^2} - \Delta_2 \right\} \times \frac{(2m)^2 \pi^2 T_c^2}{(2m)^2 \pi^2 T_c^2 + \tilde{\Omega}_p^2} \int_0^{E_F} \frac{d\xi}{(2m+1)^2 \pi^2 T_c^2 + \xi^2}; \quad (3.4)$$

$$\Delta_2 = 4\alpha T_c \sum_m \left\{ \Delta_1 \frac{\tilde{\Omega}_p^2}{(2m+1)^2 \pi^2 T_c^2 + \tilde{\Omega}_p^2} - \Delta_2 \right\} \int_0^{E_F} \frac{d\xi}{(2m+1)^2 \pi^2 T_c^2 + \xi^2}. \quad (3.5)$$

As it is easy to show, $(2m)^2$ can be replaced under the sign of the sum in (3.4) by $(2m+1)^2$, with the same accuracy. As a result, result, by changing to the dimensionless quantities

$$x = \xi / \tilde{\Omega}_p; \quad t = \pi T_c / \tilde{\Omega}_p; \quad \beta = \tilde{\Omega}_p / E_F, \quad (3.6)$$

we present the condition of solvability of system (3.4), (3.5), which plays the role of an equation for determination of T_c in the form

$$[1 + I_1(t)][1 + I_4(t)] - I_3(t)[1 + I_2(t)] = 0, \quad (3.7)$$

where

$$I_1(t) = \frac{4\alpha t}{\pi} \sum_m \frac{(2m+1)^2 t^2}{[(2m+1)^2 t^2 + 1]^2} \int_0^{1/\beta} \frac{dx}{(2m+1)^2 t^2 + x^2}; \quad (3.8)$$

$$I_2(t) = \frac{4\alpha t}{\pi} \sum_m \frac{(2m+1)^2 t^2}{(2m+1)^2 t^2 + 1} \int_0^{1/\beta} \frac{dx}{(2m+1)^2 t^2 + x^2}; \quad (3.9)$$

$$I_3(t) = \frac{4\alpha t}{\pi} \sum_m \frac{1}{(2m+1)^2 t^2 + 1} \int_0^{1/\beta} \frac{dx}{(2m+1)^2 t^2 + x^2}; \quad (3.10)$$

$$I_4(t) = \frac{4\alpha t}{\pi} \sum_m \int_0^{1/\beta} \frac{dx}{(2m+1)^2 t^2 + x^2} = 2\alpha \int_0^{1/\beta} \frac{dx}{x} \operatorname{th} \frac{\pi x}{2t}. \quad (3.11)$$

By performing a summation over m in (3.8)-(3.10), with condition $t \ll 1$ and, then, partial integration over x , we obtain: /14

$$\left. \begin{aligned} I_1 &= \alpha \left[\frac{\beta}{\beta^2 - 1} - 2L_1 \right]; \quad I_2 = \alpha \left[\frac{1}{2} \ln \left| \frac{\beta+1}{\beta-1} \right| - L_2 \right]; \\ I_3 &= 2\alpha \left[L_3 - \frac{1}{2} \ln \left| \frac{\beta+1}{\beta-1} \right| \right]; \quad I_4 = 2\alpha \ln \left(\frac{2\delta}{\beta t} \right), \end{aligned} \right\} \quad (3.12)$$

where $\ln \gamma \equiv C$ is the Euler constant, and integrals $L_{1,2,3}$ are determined in the following manner:

$$\left. \begin{aligned} L_1 &= \int_0^{1/\beta} \frac{x dx}{(1-x^2)^2} \operatorname{th} \frac{\pi x}{2t}, \quad L_2 = \int_0^{1/\beta} \frac{dx}{(1-x^2)} \operatorname{th} \frac{\pi x}{2t}; \\ L_3 &= \int_0^{1/\beta} \frac{dx}{x(1-x^2)} \operatorname{th} \frac{\pi x}{2t}. \end{aligned} \right\} \quad (3.13)$$

After partial integration, L_2 and L_3 are reduced to the integral

$$\int_0^\infty \frac{dx}{ch^2 x} \ln |x^2 - a^2| = 2 \left\{ \ln \pi + \operatorname{Re} \Psi \left(\frac{1}{2} + i \frac{a}{\pi} \right) \right\}, \quad (3.14)$$

which was calculated in [37]. (Here, Ψ is the logarithmic derivative of the gamma function.) Integral L_1 is calculated, by means of differentiation over the parameters, and it equals:

$$L_1 = \frac{\beta^2}{2(\beta^2 - 1)} + \frac{\beta}{2\pi} \operatorname{Im} \Psi' \left(\frac{1}{2} + i \frac{\beta}{\pi} \right), \quad (3.15)$$

where $B \approx \pi/2t$. With the condition $t \ll 1$; after using the known asymptotic Ψ function and its derivative, to within the terms $\sim t^2$, we obtain

$$L_1 \approx \frac{1}{2(\beta^2 - 1)}; \quad L_2 \approx \frac{1}{2} \ln \left| \frac{\beta^2}{\beta^2 - 1} \right|; \quad L_3 = L_2 + \ln \left(\frac{2\delta}{\beta t} \right). \quad (3.16)$$

Equation (3.7), with (3.12) and (3.16) taken into account, takes the form

$$\ln\left(\frac{2\gamma}{\beta t}\right) = \frac{1 + \alpha/(\beta+1) + 2\alpha R(\beta) [1 + 2\alpha R(\beta)]}{2\alpha [2\alpha R(\beta) - \alpha/(\beta+1)]} \quad (3.17)$$

where $R(\beta) = \ln(\frac{\beta+1}{\beta})$. From this, by introducing the dimensionless parameter /15

$$\sigma = \frac{4\alpha^2}{\beta} \equiv \frac{2m^*e^4}{\pi^2 \tilde{\Omega}_p \epsilon_\infty^2} \equiv \frac{\tilde{\mathcal{E}}_0}{\tilde{\Omega}_p} \quad (3.18)$$

regardless of electron concentration in the film n_0 , we obtain the following exponential formula for the critical transition temperature of the "sandwich" to the superconducting state

$$T_c = \frac{2\gamma}{\pi} \frac{\tilde{\mathcal{E}}_0}{\beta \sigma} \exp\{-1/\Lambda\}, \quad (3.19)$$

where

$$\Lambda = \frac{\beta \sigma [R(\beta) - 1/2(\beta+1)]}{1 + \sqrt{\beta \sigma}/2(\beta+1) + \sqrt{\beta \sigma} R(\beta) [1 + \sqrt{\beta \sigma} R(\beta)]} \quad (3.20)$$

We note that, with any values of parameters β and σ , the electron-plasmon interaction constant $\Lambda > 0$.

Ratio $T_c/\tilde{\mathcal{E}}_0$ vs β is presented in Fig. 1, for various values of σ , and level lines of the quantity $T_c/\tilde{\mathcal{E}}_0$ in the planes of parameters σ and β are presented in Fig. 2. (Analogous relations for T_c were obtained in [25], with a rougher approximation of the solution of equation (3.2), by a test function in the form of two steps.) As we see, the maximum value of $(T_c/\tilde{\mathcal{E}}_0)_{\max} \approx 1.9 \cdot 10^{-2}$, is reached at $\beta \approx 1.2$ and $\sigma \approx 4.2$. As an example, we consider a film with parameters $m^* \approx 0.05 m_e$ and $n_0 \approx 10^{18} \text{ cm}^{-3}$ (degenerate semiconductor) and covers with $\epsilon_\infty \approx 1.4$ and $\tilde{\Omega}_p \approx 0.05 \text{ eV}$ (semimetal), so that $K_F \approx 3 \cdot 10^6 \text{ cm}^{-1}$ /16 $E_F \approx 0.07 \text{ eV}$ and $\tilde{\mathcal{E}}_0 \approx 0.14 \text{ eV}$; in this case, $\beta \approx 0.7$ and $\sigma \approx 2.6$. Then, for T_c , according to (3.19) and (3.20), we obtain an estimate of $T_c \approx 15^\circ \text{K}$, which holds true for sufficiently thin films $d \lesssim 1/K_F \approx 3.0 \text{ nm}$.

4. Exciton Superconductivity

We proceed to consideration of the "sandwich" model which consists of a thin film of degenerate (alloy) semiconductor or semimetal, with an insulating or semiconductor coating. In such a layered system, owing to the relatively slight shielding of the Coulomb interactions, in principle, Cooper pairing of the free carriers in the film is possible, by means of exchange with

virtual collective excitations of the bound electrons (excitons) in the coverings (Ginzburg-Kirzhnits model [5,6]).

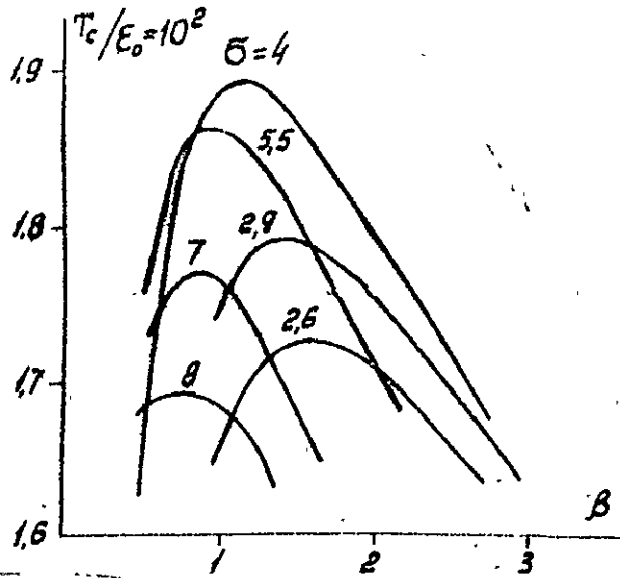


Fig. 1.

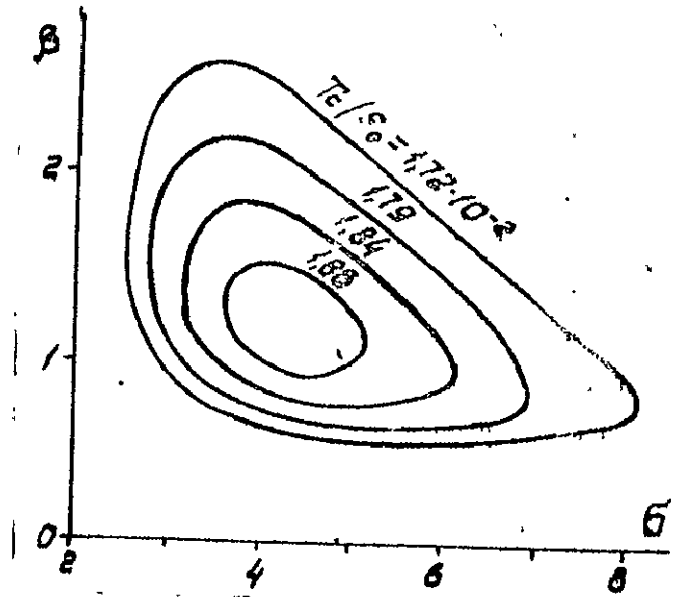


Fig. 2.

In the simplest case of a two level scheme, the dielectric permeability of the outer medium, without taking account of attenuation and spatial dispersion (the latter was taken into account in model form in [38]), has the form

/17

$$\epsilon_o(i\omega_e) = \epsilon_\infty \left[1 + \frac{\Omega_o^2 f_o}{\omega_o^2 + \omega_e^2} \right], \quad (4.1)$$

where Ω_o is the plasma frequency of the bound electrons and ω_o and f_o are the frequency and strength of the oscillator of the transition between levels. In this case, equation (2.13) is reduced to the form (compare with (3.2));

$$\bar{\Delta}(\epsilon_e) = -4\alpha T_c \sum_m \bar{\Delta}(\epsilon_m) \frac{(\epsilon_e - \epsilon_m)^2 + \omega_o^2}{(\epsilon_e - \epsilon_m)^2 + \omega_o^2 + \Omega_o^2 f_o} \times \int_0^{\epsilon_F} \frac{d\xi}{\epsilon_m^2 + \xi^2}. \quad (4.2)$$

We approximate a solution of equation (4.2) with the function

$$\bar{\Delta}(\epsilon_e) = \Delta_1 \frac{\Omega_o^2 f_o}{\epsilon_e^2 + \omega_o^2 + \Omega_o^2 f_o} - \Delta_2. \quad (4.3)$$

By substituting (4.3) in (4.2), from the boundary conditions with $\ell=0$ and $\ell \rightarrow \infty$, to within the terms $\sim (\pi T_c)^2 / \Omega_0^2 f_0$, we obtain /18

$$\Delta_1 \frac{x^2}{1+x^2} - \Delta_2 = -\frac{4\alpha Z}{\pi} \sum_m \left\{ \Delta_1 \frac{x^2}{(2m+1)^2 Z^2 + 1 + x^2} - \Delta_2 \right\} \times \frac{(2m+1)^2 Z^2 + 1}{(2m+1)^2 Z^2 + 1 + x^2} \int_0^{x/\tilde{\beta}} \frac{dy}{(2m+1)^2 Z^2 + y^2}; \quad (4.4)$$

$$\Delta_2 = \frac{4\alpha Z}{\pi} \sum_m \left\{ \Delta_1 \frac{x^2}{(2m+1)^2 Z^2 + 1 + x^2} - \Delta_2 \right\} \int_0^{x/\tilde{\beta}} \frac{dy}{(2m+1)^2 Z^2 + y^2}; \quad (4.5)$$

where

$$x = \frac{\Omega_0 \sqrt{f_0}}{\omega_0}; \quad Z = \frac{\pi T_c}{\omega_0}; \quad \tilde{\beta} = \frac{\Omega_0 \sqrt{f_0}}{E_F}; \quad y = \xi / \omega_0. \quad (4.6)$$

By performing a summation over m , we present the condition of solvability of system (4.4), (4.5), in the form (compare with (3.7)):

$$\left[J_1(z) + \frac{x^2}{1+x^2} \right] \left[1 + 2\alpha \ln \left(\frac{2\gamma x}{\tilde{\beta} Z} \right) \right] - J_3(z) [1 + J_2(z)] = 0, \quad (4.7)$$

where

$$J_1(z) = 2\alpha x^2 \int_0^{x/\tilde{\beta}} dy \left\{ \frac{1-y^2}{(1+x^2-y^2)^2} \frac{1}{y} \operatorname{th} \frac{\pi y}{2Z} - \frac{2+x^2-x^4-y^2(2+x^2)}{2(1+x^2-y^2)^2(1+x^2)^{3/2}} \right\}; \quad (4.8)$$

$$J_2(z) = 2\alpha \int_0^{x/\tilde{\beta}} dy \left\{ \frac{1-y^2}{y(1+x^2-y^2)} \operatorname{th} \frac{\pi y}{2Z} + \frac{x^2}{(1+x^2-y^2)\sqrt{1+x^2}} \right\}; \quad (4.9)$$

$$J_3(z) = 2\alpha x \int_0^{x/\tilde{\beta}} \frac{dy}{(1+x^2-y^2)} \left\{ \frac{1}{y} \operatorname{th} \frac{\pi y}{2Z} - \frac{1}{\sqrt{1+x^2}} \right\}; \quad (4.10)$$

After rather cumbersome calculations, with the use of the results obtained in Section 3, under condition $z \ll 1$, we have /19

$$J_1 = \frac{2\alpha x^2}{(1+x^2)^2} \left\{ S + \ln \left(\frac{2\gamma x}{\tilde{\beta} Z} \right) + \frac{x^3}{2(\tilde{\beta} \sqrt{1+x^2} + x)} \right\}; \quad (4.11)$$

$$J_2 = \frac{2\alpha}{1+\alpha^2} \left\{ \ln \left(\frac{2\gamma\alpha}{\beta z} \right) - \alpha^2 S \right\}; \quad (4.12)$$

$$J_3 = \frac{2\alpha\alpha^2}{1+\alpha^2} \left\{ S + \ln \left(\frac{2\gamma\alpha}{\beta z} \right) \right\}, \quad (4.13)$$

where $S = \ln \left(\frac{\beta \sqrt{1+\alpha^2}}{\alpha + \beta \sqrt{1+\alpha^2}} \right)$. From equation (4.7), with (4.11)-(4.13) taken into account, we obtain the following expression for the critical temperature.

$$T_c = \frac{2\gamma}{\pi} \frac{\tilde{\epsilon}_0}{\beta \tilde{\sigma}} \exp \left\{ -1/\tilde{\Lambda} \right\}, \quad (4.14)$$

where

$$\tilde{\Lambda} = \frac{\sqrt{\beta \tilde{\sigma}} [\alpha^2 \sqrt{\beta \tilde{\sigma}} (S - Q) - 1]}{1 + \alpha^2 [1 + \sqrt{\beta \tilde{\sigma}} (S + Q + S^2 \sqrt{\beta \tilde{\sigma}})]}; \quad (4.15)$$

$$Q = \frac{\alpha}{2(\alpha + \beta \sqrt{1+\alpha^2})}; \quad \tilde{\sigma} = \frac{4\alpha^2}{\beta} = \frac{\tilde{\epsilon}_0}{\Omega_0 \sqrt{\epsilon_0}}. \quad (4.16)$$

Formula (4.14) holds true, only in the event the electron-exciton interaction constant $\tilde{\Lambda} \neq 0$, i.e., under condition

$$\alpha^2 \sqrt{\beta \tilde{\sigma}} (S - Q) > 1, \quad (4.17)$$

which plays the part of superconductivity criterion in the exciton /20 mechanism (without consideration of phonons).

The superconductivity regions in Fig. 3 are cross hatched. They were plotted in accordance with (4.17), in the planes of parameters β and κ , for various values of $\tilde{\epsilon}_0$. As we see, as $\beta \rightarrow 0$, superconductivity is impossible with such finite values of κ and $\tilde{\epsilon}_0$. On the other hand, in the limit $\kappa \rightarrow \infty$, formula (4.14) changes to (3.19), which corresponds to a transition from bound electrons to free ($\omega_0 \rightarrow 0$).

Quantity $T_c/\tilde{\epsilon}_0$ vs. $\tilde{\epsilon}_0$ is presented in Figs. 4-6, for various values of β and κ . At $\kappa=1$ (Fig. 4), the maximum value of $T_c/\tilde{\epsilon}_0$ is a factor of ten less than $(T_c/\tilde{\epsilon}_0)_{\max}$, in plasmon superconductivity (see Fig. 1), and it decreases exponentially with decrease in κ , and it reverts to zero at the boundary of the superconductivity region. On the contrary, with increase in κ , the maximum critical temperature increases, tending towards values of T_c , which /22 are characteristic of the plasmon mechanism (see Fig. 6).

Thus, by virtue of the fact that the effective electron-exciton interaction is of a repulsive nature near the Fermi surface, exciton superconductivity proves to be less effective than plasmon.

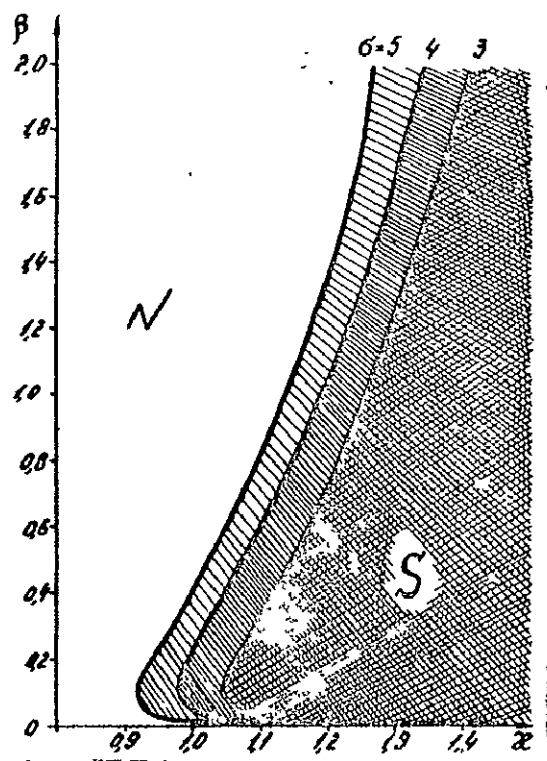


Fig. 3

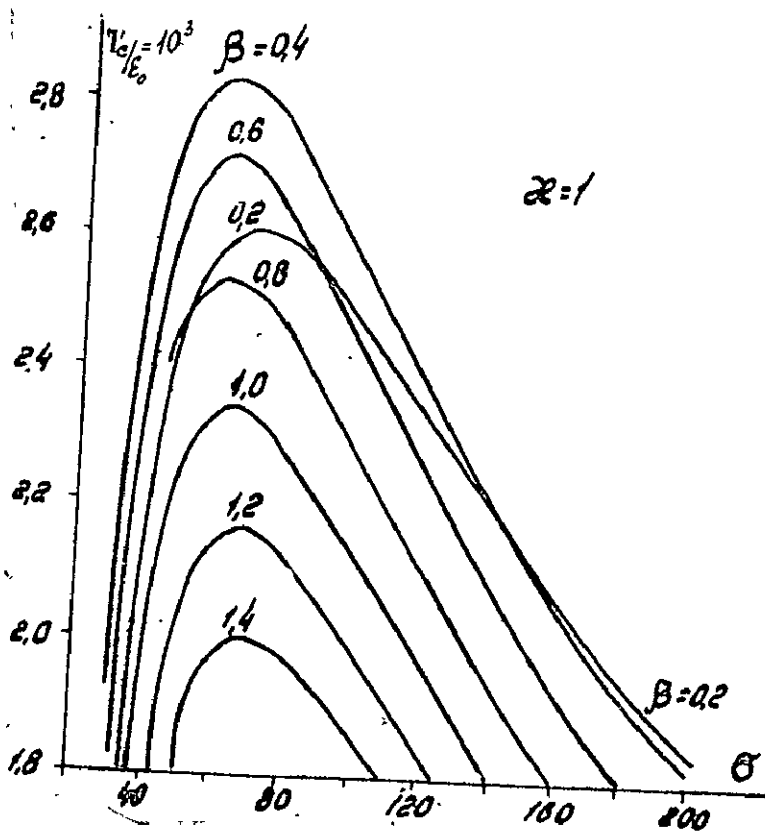


Fig. 4

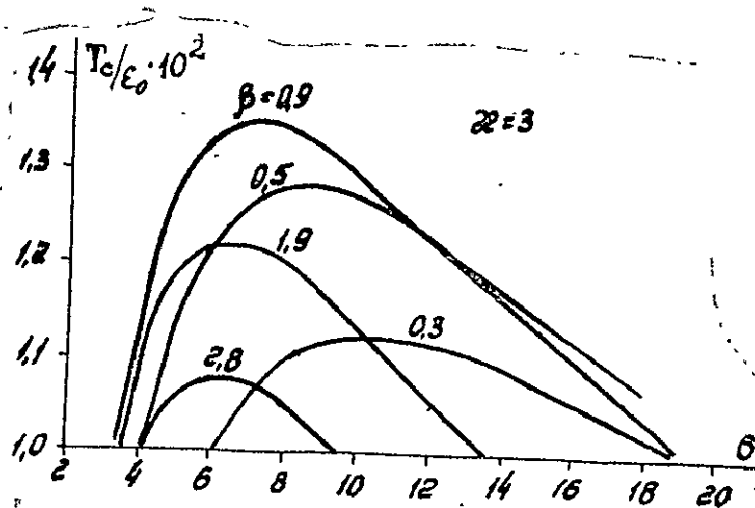


Fig. 5.

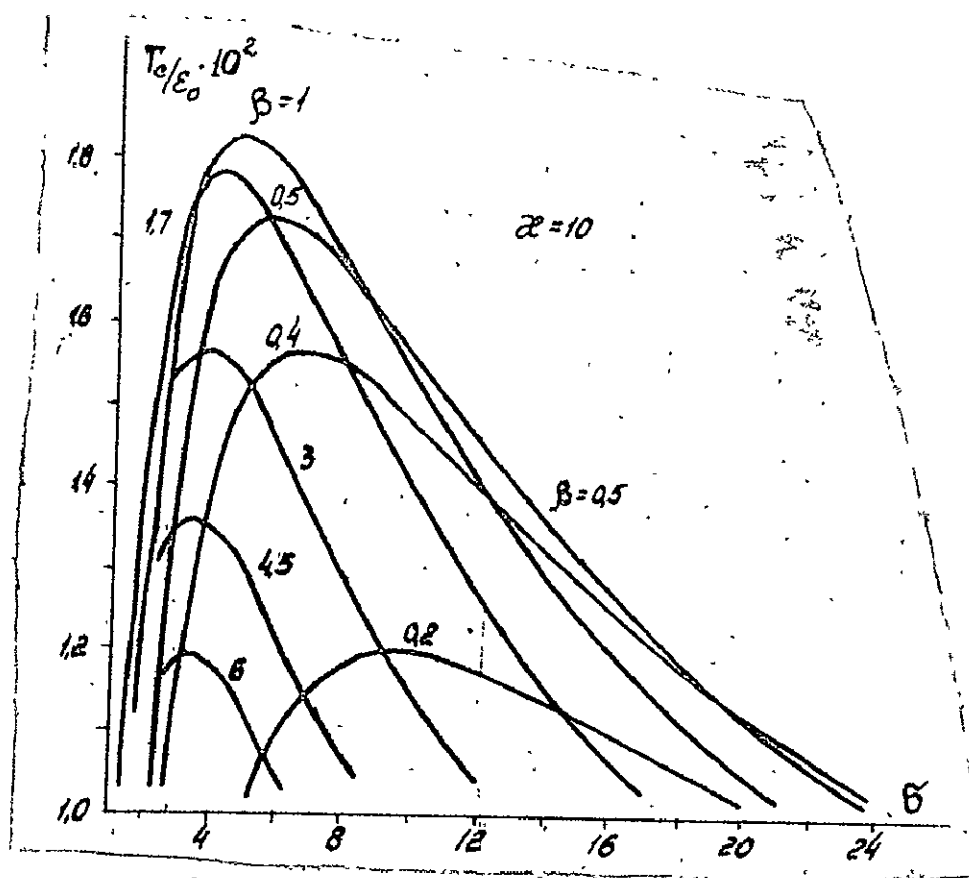


Fig. 6.

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